Czechoslovak Mathematical Journal

News and Notices

Czechoslovak Mathematical Journal, Vol. 24 (1974), No. 1, 167–169

Persistent URL: http://dml.cz/dmlcz/101228

Czechoslovak Mathematical Journal, 24 (99) 1974, Praha

NEWS and NOTICES

PROF. JAROSLAV HÁJEK AWARDED THE STATE PRIZE

On the occasion of 1st May 1973, prof. ing. dr. JAROSLAV HÁJEK, DrSc. has been awarded Klement Gottwald State Prize for his work in the asymptotic theory of statistical rank tests.

Statistical tests are a basic indispensable tool in the treatment of experimental data, or, in a still wider region — everywhere where random phenomena occur. Among these tests a distinguished role is played by the so called rank tests which are based only on the ranks of the observations ordered according to their magnitude (rather than on the values of the observations themselves). At the beginning, rank tests were considered to be only a simple, quick, but worse, substitute for classical tests developped e.g. under the assumption of the normal distribution. Later, however, statisticians found and began to admit that the normal distribution is not so often met with in the real life, and, moreover, that classical tests can very heavily change their properties if the assumptions are changed. Therefore, a much stronger attention began to be paid to rank tests, which are valid for very broad classes of distributions, e.g. for all continuous distributions, and which are robust, i.e. their properties change only little if the assumptions are little changed; furthermore, it was proved that rank tests, provided a suitable choice had been done, are very efficient compared to classical tests. As a result of all this, in the past 10—20 years rank tests penetrated into a wide spectrum of various applications, and also theoretically they belong to the most flourishing branches of mathematical statistics.

Prof. Hájek worked in particular in the asymptotic theory of rank tests and proved many important results making use of his own original, remarkable and fruitful ideas. Roughly speaking, these results concerned the asymptotic distributions of rank statistics first under the null hypothesis, then under "close", contiguous alternatives, and finally under general, non-contiguous alternatives. Let us now briefly mention some of the most important publications by prof. Hájek in this area.

Prof. Hájek started to work clear-sightedly in rank test theory very early — in his thesis in 1949 — that is at the very beginning of his scientific activity, and at the time when only rudiments of some theory in this area started to created. A part of this thesis was then published in the paper "*Some rank distributions and their applications*", Časopis pro pěstování matematiky *80* (1955), 17—31, where the generating functions are derived, and asymptotic normality is proved, of the distributions of the statistics which are now known as the Wilcoxon two-sample and the Wilcoxon one-sample statistics, and the Kendall rank correlation coefficient.

Prof. Hájek's first famous paper in this area was "*Some extensions of the Wald-Wolfowitz-Noether theorem*", Ann. Math. Statist. *32* (1961), 506—523, where he found a necessary and sufficient condition for the asymptotic normality of general linear rank statistics under the null hypothesis, i.e. under the assumption that the vector of ranks takes on all permutations with equal probabilities. This theorem generalized a number of previous results of other authors; it is now commonly quoted under Hájek's name, or under the names Wald-Wolfowitz-Noether-Hájek. The original method of the paper is based on the idea that a rank statistic S_n is asymptotically equivalent in the quadratic mean to a suitable sum T_n of independent asymptotically small random variables (exactly that $\lim_{n\to\infty} E(S_n - T_n)^2/\mathrm{var}\, T_n = 0$).

Further important paper is "*Asymptotically most powerful rank-order tests*", Ann. Math. Statist. *33* (1962), 1124—1147, where the asymptotic normality of linear rank statistics for testing regression is proved under contiguous alternatives, the asymptotic efficiency of the respective tests is investigated, and the form of the asymptotically most powerful test is found. In addition, a universally asymptotically most powerful rank test is constructed here; its existence was a surprise for specialists at that time. With regard to method, a remarkable contribution of the paper is the use of the notion of contiguity, which had been introduced by LeCam originally for another purpose. (In general, if P_n, Q_n are two sequences of probability measures, we say that Q_n is contiguous to P_n if, for any sequence of events A_n, $P_n(A_n) \to 0$ implies $Q_n(A_n) \to 0$. For the regression model under discussion this means that the sequence of alternatives approaches the null hypothesis, i.e. that certain conditions are required for the limit behaviour of the regression coefficients.)

Another interesting Hájek's idea was a representation of the well-known Kolmogorov-Smirnov statistic in a different equivalent form by means of so called antiranks, which enabled him then to generalize this statistic in a natural way also for regression alternatives. This was done in the paper "*Extension of the Kolmogorov-Smirnov test to regression alternatives*", Bernoulli-Bayes-Laplace Anniversary Volume, Springer Verlag 1965, pp. 45—60. Moreover, it is proved here that such a generalized statistic converges in distribution to the Brownian bridge.

A general systematic theory of rank tests together with a number of concrete special cases were then presented in a monograph "*Theory of rank tests*", Academia, Prague & Academic Press, New York—London, 1967, which was written by prof. J. Hájek jointly with Z. Šidák. (In 1971, the publishing house Nauka in Moscow published also its Russian translation.) In Chapters V, VI, VII of this book, written by prof. Hájek, his previous results in the asymptotic theory are gathered, of course in a more systematic, complete and improved manner. Among important new results in the book it is particularly worthwhile to mention the proof of the asymptotic sufficiency of the vector of ranks and its consequences in § VII.1.

Since the above monograph was written on a rather advanced mathematical level, prof.Hájek published another, easier, book "*A course in nonparametric statistics*", Holden-Day, San Francisco 1969, intended for students for their first steps in the theory of rank tests. (Its Russian translation is being prepared.)

More recently, prof. Hájek proved many interesting results on the asymptotic normality of rank statistics under general, non-contiguous alternatives in the papers "*Asymptotic normality of simple linear rank statistics under alternatives*", Ann. Math. Statist. *39* (1968), 325—346, "*Asymptotic normality of simple linear rank statistics under alternatives* II", Ann. Math. Statist. *40* (1969), 1992—2017, "*Asymptotic normality of the Wilcoxon statistic under divergent alternatives*", Zastosowania matematyki *10* (1969), 171—178 (the latter two papers jointly with V. Dupač). The methods used in these papers are elementary in its essence but much involved and ingenious; they are based on a certain new inequality for variances of rank statistics and on an approximation of these statistics by means of their projections on the sums of independent variables.

Prof. Hájek is one of the foremost specialists in mathematical statistics, known in the world for his works not only in the theory of rank tests but also in several further areas, e.g. the theory of sample surveys, statistical inference in random processes, philosophical and logical foundations of mathematical statistics and others. His works contain many important and fruitful ideas, at the time of their publication they meant a considerable step ahead in the development of the respective area, and their results were quoted and pursued further by many Czechoslovak and foreign authors. Several fundamental theorems are usually quoted under Hájek's name: we have mentioned above the theorem on the asymptotic normality of rank statistics, further instances are Feldman-Hájek's theorem saying that the probability measures of two normal processes are either mutually equivalent or singular, and Hájek-Rényi's inequality for the sums of independent variables.

The scientific activity of prof. Hájek is closely connected with applications of mathematical statistics. Prof. Hájek himself directly collaborated in many applied problems in diverse fields, and, in addition, also his theoretical results have a considerable and rather immediate importance for applications. Let us mention only the theory of rank tests: first, his results on the asymptotic distributions are used in practice for cases with a large number of observations, second, prof. Hájek also investigated the forms of the optimal tests in different situations and their appropriate choice, and proposed some new tests.

Besides his own scientific work, prof. Hájek devotes himself very intensively to the education of younger scientific workers and students in the Department of Mathematical Statistics at the Faculty of Mathematics and Physics of the Charles University, and he has raised to a high level all the work in this Department. Under his guidance a number of workers received their Candidate of Science degrees, and on the basis of his suggestions and advice many further interesting works of his students have been written (let us mention again only the works in the theory of rank tests by J. Anděl, M. Hušková, J. Jurečková, D. Vorlíčková and a Vietnamese student Nguyen-van Ho).

On behalf of all readers of this Journal we congratulate prof. J. Hájek upon the award of the State Prize, and we trust that his health will improve so as to enable him to continue his work and to enrich mathematical statistics by many further results and ideas of his.

Zbyněk Šidák, Praha

ANNOUNCEMENT

The Second Czechoslovak Symposium on Graph Theory will be held in Prague, Czechoslovakia, from June 24 to June 28, 1974. Chairman of the Organizing Commitee: Professor Miroslav Fiedler, Mathematical Institute of the Czechoslovak Academy of Sciences, Žitná 25, 115 67 Praha 1, Czechoslovakia.