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BOOK REVIEWS

THEORY OF NONLINEAR OPERATORS, Proceedings of a Summer School held in September 1971 at Babylon, Czechoslovakia. Academia, Publishing House of the Czechoslovak Academy of Sciences, Prague 1973. Reviewers: J. Kopáček, A. Kufner. Kčs 52,—.

In the late 1960's the study of the properties of nonlinear mappings of an infinite-dimensional Banach space X into another such space Y, as well as the problems of existence, uniqueness, and multiplicity of the solutions of equations involving such mappings, was extremely popular. This topic has been studied among other countries also in Czechoslovakia, which is why the main subject of the summer-school held from 13 to 17 September 1971 at Babylon (organized by the Mathematical Institute of the Czechoslovak Academy of Sciences) was the analysis of nonlinear operators and its applications — particularly in the theory fo boundary value problems for differential equations; nevertheless other problems connected with the solvability of nonlinear operator equations were considered as well.

The papers presented in this volume reflect the above-mentioned subjects of functional analysis.

In the longest paper of this volume, the paper by H. TRIEBEL (*Interpolation for spaces of Besov type. Elliptic differential operators*, pp. 135—191), a summary of some results concerning interpolation properties, inclusion properties, isomorphic properties as well as the existence of Schauder bases and embedding and extension theorems is given for spaces of Besov type. The author considers function spaces without weights and with weights defined in a domain (bounded or unbounded) in the Euclidean n-space including the Sobolev, Slobodeckij, Besov, Lebesgue spaces and also the spaces of distributions. Some applications to regular and singular elliptic differential operators are given. The results (41 theorems) are presented without proofs and the reader is referred to the bibliography (66 numbers).

Theorems about global and local surjectivity for monotone, demicontinuous and coercive operators are found in S. SPAGNOLO: *On the solvability of some non-linear equations*, pp. 127—133.

E. S. TSITLANADZE (*Quelques questions de l'analyse mathematique dans les espaces localement lineaires*, pp. 193—207) desires to investigate nonlinear operators and functionals defined not on subsets of linear spaces, but on subsets of a "nonlinear space". To this end he introduces locally linear spaces and proves in such spaces results which are analogous to the well-known theorems about weakly continuous functionals in Banach spaces. No useful example is given.

The paper by M. S. BERGER (*On nonlinear spectral theory*, pp. 29—37) is a study of the structure of solutions of an operator equation $F(x, \lambda) = 0$ (depending explicitly on a parameter λ) as the parameter λ varies. Applications to quasilinear elliptic partial differential equations and to periodic solutions for autonomous dynamical systems are shown.

R. KLUGE (*Folgen und Iterationsverfahren bei Folgen nichtlinearer Variationsungleichungen*, pp. 39—47, *Zur numerischen Approximation von nichtlinearen Eigenwertproblemen und Fixpunktbifurkationen für mehrdeutige Abbildungen*, pp. 49—55) gives a survey (without proofs) of results about iterative, projective and iterative-projective methods for solving nonlinear equations and inequalities which have been compiled from approximately 25 papers of the author.

A very interesting result is proved in the paper by A. AMBROSETTI and G. PRODI (*On the inversion of some differentiable mappings with singularities between Banach spaces*, pp. 9—28). The abstract result may best be ilustrated on the following example. Consider the boundary value problem

$\Delta u + f(u) = g$ on Ω, $u = 0$ on the boundary of Ω. Under some assumptions on the function f there exists in the space $C^{0,\alpha}(\bar{\Omega})$ a closed connected C^1-manifold M of codimension 1, such that $C^{0,\alpha}(\bar{\Omega}) \setminus M$ consists exactly of two connected components A_1, A_2 with the following properties:

(a) if $g \in A_1$ then the considered boundary value problem has no solution;
(b) if $g \in A_2$ then the boundary value problem has exactly two solutions.

Moreover, if $g \in M$ then the boundary value problem has a unique solution.

It is interesting to observe that for such problem the method of Leray-Schauder degree yields no useful result, since the topological degree of the mapping considered is zero. It seems that this is the first nontrivial example of the so-called normal solvable nonlinear equations (i.e., the range of the nonlinear operator is closed) since the range of such operator is not the whole space $C^{0,\alpha}$.

In the paper by J. Nečas (*Fredholm theory of boundary value problems for nonlinear ordinary differential operators*, pp. 85—119) the analogue of Fredholm alternative for nonlinear operators is proved. As in the linear case it is proved that the equation $\lambda\, T(u) - S(u) = f$ (T, S are nonlinear operators satisfying certain conditions, λ is a real parameter) is solvable for each right hand side f provided the homogeneous equation $\lambda\, T(u) - S(u) = 0$ has only the trivial solution. From this we can see how important it is to study the structure of the set Λ of all eigenvalues of the couple T, S ($\lambda \in \Lambda$ iff the equation $\lambda\, T(u) - S(u) = 0$ has a nontrivial solution). The Ljusternik-Schnirelmann theory which says that the set Λ contains a sequence of positive numbers converging to zero is explained in Chapter 2. (The paper by J. Naumann (*Remarks on nonlinear eigenvalue problems*, pp. 61—84) is also devoted to this theory). Abstract results are applied to the theory of boundary value problems for ordinary differential equations. For a special type of such equations "the converse of Ljusternik-Schnirelmann theory" is proved, i.e., it is shown that the set Λ is precisely a sequence of positive numbers converging to zero. Such result was also proved for another class of operators (see Introduction), the proof being based on the Morse-Sard theorem about the size of the image of the set of all critical points of a real function — see papers by M. Kučera (*Morse-Sard theorem for functions from the class $C^{k,\alpha}$*, pp. 57—59) and by J. Souček-V. Souček (*The Morse-Sard theorem for real-analytic functions*, pp. 121—125).

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