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Czechoslovak Mathematical Journal, Vol. 16 (1966), No. 3, 377–379

Persistent URL: http://dml.cz/dmlcz/100739

INFINITESIMAL DEFORMATIONS OF SURFACES IN E^3 *)

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(Received July 5, 1965)

The Killing vector field associated to a differentiable deformation of a surface in E^3 cannot be normal even locally, the mean curvature being not equal to zero. We shall see that, for a compact surface, it cannot be too "near" to the normal vector field.

Let $S \to E^3$ be a surface of class C^3 in the Euclidean 3-space E^3 described by the point $r = r(u)$; denote by $n(u)$ the normal unit vector to S at the point $r(u)$. In a suitable domain of S, we may write

$$r = r(u^1, u^2)\,; \tag{1}$$

let $g_{ij}(b_{ij})$ be the first (second) fundamental tensor. If x^i is any vector field on S, define $x^{i;j} = g^{jk}x^i{}_{;k}$. The following result due to K. YANO is well known: *Let S be compact. Then for each vector field x^i on S, we have*

$$\int_S (K g_{ij} x^i x^j + x^{i;j} x_{j;i} - x^i{}_{;i} x^j{}_{;j})\, d\sigma = 0\,. \tag{2}$$

This assertion remains valid for any S if

$$x^i{}_{;j} x^j - x^j{}_{;j} x^i = 0 \tag{3}$$

outside a compact subset of S.

Let $S \times R \to E^3$ be a one-parametric family of isometric surfaces in E^3; suppose that $S(0)$ is the surface S. Let us denote by w the vector field

$$w = \frac{\partial r(u, t)}{\partial t} \tag{4}$$

in E^3; let

$$V = v^i r_i + vn \tag{5}$$

*) This work was partly supported by the National Science Foundation through research projects at Brandeis University (Waltham, Mass., U.S.A.).

be the restriction of the field (5) to the surface S. We have

$$v_{i;j} + v_{j;i} = 2vb_{ij} . \tag{6}$$

Each vector field (5) on S satisfying (6) is called the *Killing vector field*. For each Killing vector field, we have

$$v^{i}_{\ ;i} = vH . \tag{7}$$

For a Killing vector field (5), the integral formula (2) reduces to

$$\int_S (Kg_{ij}v^i v^j + 2vb_{ij}v^{i;j} - v^{i;j}v_{i;j} - v^2H^2)\, \mathrm{d}\sigma = 0 . \tag{8}$$

We are now in the position to prove

Theorem 1. *Let* (5) *be a Killing vector field on the surface* (which is not necessary to be compact) S, *and suppose*: (a) $V = 0$ *outside some compact subset* $C \subset S$, (b) *the Gauss curvature of* S *is non-positive inside a domain* $D \supset C$, (c) *we have*

$$b_{ij}v^{i;j} \geqq 0 \tag{9}$$

on S. *If* $v \leqq 0$ *on* S, *then*

$$v^{i}_{\ ;j} = 0 \tag{10}$$

on S *and* (i) $V_T \equiv v^i r_i = 0$ *at each point with* $K < 0$, (ii) $V = V_T$ *at each point with* $H \neq 0$.

Theorem 2. *Let* S *be a compact surface with non-vanishing mean curvature. Let* (5) *be a Killing vector field on* S *such that*

$$b^2 + H^2Kl \geqq 0 \quad on \quad S , \tag{11}$$

where

$$b = b_{ij}v^{i;j}, \quad l = g_{ij}v^i v^j . \tag{12}$$

Further, let

$$v \geqq \frac{b + \sqrt{(b^2 + H^2Kl)}}{H^2} \quad or \quad v \leqq \frac{b - \sqrt{(b^2 + H^2Kl)}}{H^2} \quad on \quad S . \tag{13}$$

Then we have (10) *and*

$$v^2 = \frac{Kl}{H^2} . \tag{14}$$

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Резюме

ИНФИНИТЕЗИМАЛЬНОЕ ИЗГИБАНИЕ ПОВЕРХНОСТЕЙ В E^3

АЛОИС ШВЕЦ (Alois Švec), Прага

Пусть $r = r(u; t)$ – однопараметрическая система $S(t)$ поверхностей в E^3. Пусть

$$V = v^i \frac{\partial r(u; 0)}{\partial u^i} + v \,.\, n(u; 0)$$

является ограничением векторного поля $w = \partial r(u; t)/\partial t$ на поверхность $S(0)$; $n(u; 0)$ означает единичное нормальное векторное поле поверхности $S(0)$. Если $S(t)$ – система изометрических поверхностей, то выполнено (6), где b_{ij} – второй тензор поверхности $S(0)$. Далее, справедлива интегральная формула (8), на основании которой высказаны две теоремы.