Czechoslovak Mathematical Journal

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Czechoslovak Mathematical Journal, Vol. 16 (1966), No. 1, 63–69

Persistent URL: http://dml.cz/dmlcz/100710

Чехословацкий математический журнал, т. 16 (91) 1966, Прага

ON SEMIGROUPS WHICH ARE UNIONS OF COMPLETELY 0-SIMPLE SUBSEMIGROUPS

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(Received December 12, 1964)

1.

The Wedderburn-Artin and the Noether Structure Theorems give satisfactory characterizations of semisimple associative rings. In the paper KERTÉSZ-STEINFELD [4] there are given some other characterizations of these rings. Rees' well known Theorem for completely 0-simple semigroups plays the same rôle as the second Wedderburn-Artin Theorem does for simple rings. In his paper [5] SCHWARZ studied among others the semigroups without proper radical which are unions of their 0-minimal left ideals. The purpose of this paper is to give some equivalent conditions for semigroups with zero which are analogous to the first Wedderburn-Artin Theorem, Noether Theorem and other characterizations of semisimple rings. We shall prove that for a semigroup S with zero the following conditions are equivalent:

S is a 0-direct union of two-sided ideals which are completely 0-simple subsemigroups of S;

S is regular and the union of its 0-minimal left ideals;

S is regular and the union of its 0-minimal quasi-ideals. (See Theorem 15.)

This characterization is in a close connection with Chapter 6 of CLIFFORD-PRESTON'S book [3]. The basic ideas for this Chapter are to be found in Schwarz's paper [5] and a great deal of it is devoted to theorems of this type.

In section 2 we mention Lemmas and Theorems needed in the proof of Theorem 15 and Corollary 16.

2.

We use the terminology of Clifford-Preston's book [2] and we cite some results from it without proof.

Theorem 1. ([2], Theorem 2.48.) *Let S be a 0-simple semigroup. Then S is completely 0-simple if and only if it contains at least one 0-minimal left ideal and at least one 0-minimal right ideal.*

Corollary 2. ([2], Corollary 2.49.) *A completely* 0-*simple semigroup is the union of its* 0-*minimal left* (*right*) *ideals.*

Theorem 3. ([2], Theorem 2.51.) *A completely* 0-*simple semigroup is regular.*

Theorem 4. ([2], Theorem 1.17.) *The following two conditions on a semigroup S are equivalent*:

(i) *S is regular, and any two idempotents of S commute with each other*;
(ii) *S is an inverse semigroup* (*i.e. every element of S has a unique inverse in S*).

Theorem 5. ([2], Theorem 3.9.) *The following conditions on a semigroup S with* 0 *are equivalent*:

(α) *S is a completely* 0-*simple inverse semigroup*;
(β) *S is a Brandt semigroup.*

A subset $\mathfrak{k} \neq \square$ of a semigroup S is called a *quasi-ideal* of S if $\mathfrak{k}S \cap S\mathfrak{k} \subseteq \mathfrak{k}$.

We have

Lemma 6. (Cf. [8], Lemma 2.) *Let e be an idempotent element,* $\mathfrak{l}$ *a left ideal,* $\mathfrak{r}$ *a right ideal of a semigroup S with* 0. *Then* $\mathfrak{r} \cap \mathfrak{l}$, $e\mathfrak{l}$ *and* $\mathfrak{r}e$ *are quasi-ideals of S.*

Theorem 7. ([8], Satz 6.) *Let* $\mathfrak{l}$ *be a* 0-*minimal left ideal,* $\mathfrak{r}$ *a* 0-*minimal right ideal of a semigroup S with* 0. *If* $(e \neq 0)$ *is an idempotent element of* $\mathfrak{l}$ (or $\mathfrak{r}$), *then* $e\mathfrak{l}$ (*or* $\mathfrak{r}e$) *is a* 0-*minimal quasi-ideal of S.*

Theorem 8. ([8], Satz 1.) *Let* $\mathfrak{r}$ *be a* 0-*minimal right ideal and* $\mathfrak{l}$ *a* 0-*minimal left ideal of a semigroup S with* 0. *Then the meet* $\mathfrak{r} \cap \mathfrak{l}$ *is either zero or a* 0-*minimal quasi-ideal of S.*

It is easy to prove the following assertion.

Lemma 9. *Every left* (*right*) *ideal* $\neq 0$ *of a regular semigroup S with* 0 *contains at least one idempotent element* $\neq 0$.

Proof. Let $\mathfrak{l}$ be a left ideal of S; $a \in \mathfrak{l}$, $a \neq 0$. By regularity there is an $x \in S$ such that $a = axa$. Clearly $xa \neq 0$ is an idempotent and $xa \in Sa \subseteq S\mathfrak{l} \subseteq \mathfrak{l}$.

Analogously for the right ideals of S.

Let A be any subset of the semigroup S with zero. We shall say that A is *nilpotent* if for some integer $k \geqq 1$ the relation $A^k = 0$ holds. The union of all nilpotent left ideals of S is called the *radical* of S. (Cf. [5], Definition 3.2.).

Lemma 10. *If S is a semigroup with radical* 0 *then* 0 *is the unique nilpotent right ideal of S.*

Proof. If $\mathfrak{r} \neq 0$ were a nilpotent right ideal of S then (the two-sided ideal) $\mathfrak{r} \cup S\mathfrak{r} \neq 0$ would be a nilpotent ideal of S.

Lemma 9 implies the following two corollaries.

Corollary 11. *A regular semigroup* S *with* 0 *has zero radical.*

Corollary 12. *Every* 0-*minimal left (right) ideal* $\mathfrak{l}$ $(\mathfrak{r})$ *of a regular semigroup* S *with* 0 *is of the form* $\mathfrak{l} = Se$ $(\mathfrak{r} = fS)$ *with* $e^2 = e$ $(f^2 = f)$.

We shall prove two results which are analogous to two known theorems in ring theory.

Theorem 13. (Cf. Artin-Nesbitt-Thrall [1], Corollary 5.4.B.) *Let* S *be a semigroup with radical* 0. *Then* Se $(e^2 = e)$ *is a* 0-*minimal left ideal if and only if* eS *is a* 0-*minimal right ideal of* S.

Proof. Let Se $(e^2 = e)$ be a 0-minimal left ideal of S; then, in view of Theorem 7, eSe is a 0-minimal quasi-ideal of S. Let $\mathfrak{r}$ denote a right ideal of S with $0 \subset \mathfrak{r} \subseteqq eS$. Hence $e\mathfrak{r} = \mathfrak{r}$. Lemma 10 implies that $e\mathfrak{r} \,.\, e\mathfrak{r} = \mathfrak{r}^2 \neq 0$ and thus $e\mathfrak{r}e \neq 0$. By Lemma 6, $e\mathfrak{r}e$ $(\subseteqq eSe)$ is a quasi-ideal of S and so $e\mathfrak{r}e = eSe$ holds. Hence $e \in e\mathfrak{r}e \subseteqq e\mathfrak{r} = \mathfrak{r}$ which implies $eS \subseteqq \mathfrak{r}$. Therefore $\mathfrak{r} = eS$; q.e.d.

Theorem 14. (Cf. [7], Satz 7.) *Let* S *be a semigroup with radical* 0. *Then we can write every* 0-*minimal quasi-ideal* $\mathfrak{k}$ *of* S *in the form* $\mathfrak{k} = \mathfrak{l} \cap \mathfrak{r}$, *where* $\mathfrak{l}$ *is a* 0-*minimal left ideal and* $\mathfrak{r}$ *is a* 0-*minimal right ideal of* S.

Proof. The 0-minimality of the quasi-ideal $\mathfrak{k}$ and Lemma 6 imply $S\mathfrak{k} \cap \mathfrak{k}S = 0$ or $\mathfrak{k}$.

Let the first case be assumed. If $S\mathfrak{k} \leqq 0$, then $\mathfrak{k}$ is a left ideal of S with $\mathfrak{k}^2 = 0$, which is impossible. If $S\mathfrak{k} \neq 0$, then since $\mathfrak{k}S\mathfrak{k} \subseteqq S\mathfrak{k} \cap \mathfrak{k}S = 0$,

$$S\mathfrak{k} \,.\, S\mathfrak{k} = 0 \quad (S\mathfrak{k} \neq 0)$$

holds, which contradicts the assumption concerning the radical.

Thus $S\mathfrak{k} \cap \mathfrak{k}S = \mathfrak{k}$ must hold. It is sufficient to prove that $S\mathfrak{k}$ is a 0-minimal left ideal of S. Let $\mathfrak{l}$ be a left ideal of S satisfying

$$0 \subset \mathfrak{l} \subseteqq S\mathfrak{k}\,. \tag{1}$$

The relation $S\mathfrak{l} \cap \mathfrak{k}S \subseteq S\mathfrak{k} \cap \mathfrak{k}S \subseteqq \mathfrak{k}$, the 0-minimality of $\mathfrak{k}$ and Lemma 6 imply that either

$$S\mathfrak{l} \cap \mathfrak{k}S = 0 \tag{2}$$

or

$$S\mathfrak{l} \cap \mathfrak{k}S = \mathfrak{k} \tag{3}$$

holds. From (2) we obtain $\mathfrak{k}\mathfrak{l} \subseteqq S\mathfrak{l} \cap \mathfrak{k}S = 0$, i.e., $\mathfrak{k}\mathfrak{l} = 0$. Hence $S\mathfrak{k} \,.\, \mathfrak{l} = 0$, whence, in view of (1),

$$\mathfrak{l}^2 \leqq 0$$

follows. This is a contradiction to our hypothesis. From (3) we get $\mathfrak{k} \subseteqq S\mathfrak{l} \subseteqq \mathfrak{l}$. Hence $S\mathfrak{k} \subseteqq S\mathfrak{l} \subseteqq \mathfrak{l}$. This and (1) imply $\mathfrak{l} = S\mathfrak{k}$ and $S\mathfrak{k}$ is a 0-minimal left ideal of S, q.e.d.

We shall say that the semigroup S with 0 is the 0-*direct union* of its ideals $\mathfrak{a}_\alpha$ $(\alpha \in \Lambda)$ if $S = \bigcup_{\alpha \in \Lambda} \mathfrak{a}_\alpha$ and $\mathfrak{a}_\alpha \cap (\bigcup_{\alpha \neq \beta \in \Lambda} \mathfrak{a}_\beta) = 0$ hold.

The quasi-ideals $\mathfrak{k}_{\lambda\lambda'}$ $(\lambda, \lambda' \in \Lambda)$ of a semigroup S with 0 are said to form a *complete system*, if the following three conditions hold:

1) $\mathfrak{k}_{\lambda\lambda'} = 0$ or $\mathfrak{k}_{\lambda\lambda'}$ is a 0-minimal quasi-ideal of S,

2) if $\mathfrak{k}_{\lambda\lambda'} \neq 0$, then it is of the form $e_\lambda S e_{\lambda'}$ for some idempotents e_λ, $e_{\lambda'}$ $(\lambda, \lambda' \in \Lambda)$,

3) $\mathfrak{k}_{\lambda\lambda'} \neq 0$ implies $\mathfrak{k}_{\lambda'\lambda}\mathfrak{k}_{\lambda\lambda'} \neq 0$ $(\lambda, \lambda' \in \Lambda)$.

This notion is analogous to the notion of the complete system of quasi-ideals introduced by KERTÉSZ-STEINFELD [4] for associative rings.

3.

Theorem 15. *The following conditions on a semigroups S with 0 and with more than one element are equivalent*:

(A) *S is regular and the union of its 0-minimal left ideals*;

(B) *S is a union of 0-minimal left*[1] *ideals of the form Se_λ $(e_\lambda^2 = e_\lambda;\ \lambda \in \Lambda)$*;

(C) *S is a 0-direct union of two-sided ideals*[2] *which are completely 0-simple subsemigroups of S*;

(D) *S is a union of quasi-ideals which form a complete system*;

(E) *S is regular and the union of its 0-minimal quasi-ideals.*

Proof. (A) implies (B). In view of Corollary 12 every 0-minimal left ideal $\mathfrak{l}_\lambda$ of S is of the form $\mathfrak{l}_\lambda = Se_\lambda$, where e_λ is an idempotent $\in \mathfrak{l}_\lambda$.

(B) implies (C)[3]. First, we show that the radical of S is 0. Let se_λ $(\neq 0)$ $(se_\lambda \in Se_\lambda)$ be an arbitrary element of the ideal $\mathfrak{m}$ $(\neq 0)$ of S. This implies $\mathfrak{m} \cap Se_\lambda \neq 0$. With respect to the 0-minimality of the left ideal Se_λ it must hold $e_\lambda \in Se_\lambda \subseteq \mathfrak{m}$. Thus $\mathfrak{m}$ cannot be nilpotent and the radical of S is indeed 0.

As the left ideals Se_λ $(\lambda \in \Lambda)$ are 0-minimal

$$\text{(4)} \qquad \text{either } Se_\lambda Se_{\lambda'} = 0 \text{ or } Se_\lambda Se_{\lambda'} = Se_{\lambda'}$$

holds. It is easy to see that the relations $\equiv$ defined by

$$\text{(5)} \qquad Se_\lambda \equiv Se_{\lambda'} \Leftrightarrow Se_\lambda Se_{\lambda'} = Se_{\lambda'}$$

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[1]) If in conditions (A), (B) "0-minimal left ideals" is replaced by "0-minimal right ideals", one obtains conditions equivalent to the original (A)—(E).

[2]) In Section 3 of his paper [6] Schwarz proves some similar decomposition theorems for dual semigroups with radical 0.

[3]) One can prove this part with the help of the theorems in Section 9 of Schwarz [5].

is an equivalence relation in the set of the 0-minimal left ideals $Se_\lambda (\lambda \in \Lambda)$. Let $\mathfrak{a}_\alpha$ denote the union of all the left ideals belonging to the equivalence class K_α.

Thus

$$S = \bigcup_{\alpha \in A} \mathfrak{a}_\alpha \tag{6}$$

where A denotes the index set of the different classes.

First, we show that $\mathfrak{a}_\alpha = \bigcup_{Se_\mu \in K_\alpha} Se_\mu$ is a 0-simple two-sided ideal of S. (4) and (5) imply

$$\mathfrak{a}_\alpha \mathfrak{a}_\beta = \begin{cases} \mathfrak{a}_\alpha & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta . \end{cases} \tag{7}$$

Hence in view of (6) it follows

$$S\mathfrak{a}_\alpha = \mathfrak{a}_\alpha S = \mathfrak{a}_\alpha \tag{8}$$

Let $\mathfrak{b}$ be an ideal of $\mathfrak{a}_\alpha$. With respect to (6) and (7) $\mathfrak{b}$ is an ideal of S and therefore $\mathfrak{b}^2 \neq 0$ holds. Hence

$$0 \neq \mathfrak{b}^2 \subseteqq \mathfrak{b}\mathfrak{a}_\alpha = \bigcup_{Se_\mu \in K_\alpha} \mathfrak{b} Se_\mu . \tag{9}$$

This implies $\mathfrak{b}Se_\nu \neq 0$ for some $Se_\nu \in K_\alpha$, whence $\mathfrak{b}Se_\nu = Se_\nu \subseteqq \mathfrak{b}$ follows. In view of (5) we obtain

$$\mathfrak{b} \supseteqq \mathfrak{b}\mathfrak{a}_\alpha \supseteqq Se_\nu \mathfrak{a}_\alpha = \bigcup_{Se_\mu \in K_\alpha} Se_\nu Se_\mu = \bigcup_{Se_\mu \in K_\alpha} Se_\mu = \mathfrak{a}_\alpha$$

establishing the 0-simplicity of $\mathfrak{a}_\alpha$.

As $\mathfrak{a}_\alpha$ is a 0-simple ideal of S,

$$\mathfrak{a}_\alpha \cap \Big(\bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\beta \Big) = 0 \text{ or } \mathfrak{a}_\alpha$$

holds. The second case implies $\mathfrak{a}_\alpha \subseteqq \bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\beta$. By multiplication by $\mathfrak{a}_\alpha$ we obtain in view of (7)

$$\mathfrak{a}_\alpha^2 \subseteqq \mathfrak{a}_\alpha \Big(\bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\beta \Big) = \bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\alpha \mathfrak{a}_\beta = 0 ,$$

which is a contradiction. Thus for every $\mathfrak{a}_\alpha$ $(\alpha \in A)$

$$\mathfrak{a}_\alpha \cap \Big(\bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\beta \Big) = 0$$

must hold and therefore (6) is a 0-direct union.

If Se_μ is a 0-minimal left ideal contained in $\mathfrak{a}_\alpha$, then $e_\mu S$ is a 0-minimal right ideal in $\mathfrak{a}_\alpha$. By Theorem 1 $\mathfrak{a}_\alpha$ is a completely 0-simple subsemigroup of S.

(C) implies (D). Let S be the 0-direct union of its ideal $\mathfrak{a}_\alpha$ $(\alpha \in A)$, where $\mathfrak{a}_\alpha$ are completely 0-simple subsemigroups of S. In view of Theorem 3 $\mathfrak{a}_\alpha$ $(\alpha \in A)$ are regular semigroups, therefore S is itself regular. Corollary 2 implies that every $\mathfrak{a}_\alpha$ $(\alpha \in A)$ is the union of its 0-minimal left ideals and the union of its 0-minimal right ideals. Since S is the 0-direct union of the 0-simple ideals $\mathfrak{a}_\alpha$ $(\alpha \in A)$, all the left (right) ideals of $\mathfrak{a}_\alpha$ $(\alpha \in A)$ are left (right) ideals of S. Thus S is the union of its 0-minimal left ideals and the union of its 0-minimal right ideals.

The regularity of S implies that the 0-minimal left ideals of S are of the form Se_λ with idempotent elements $e_\lambda \neq 0$ $(\lambda \in \Lambda)$. From Corollary 11 and Theorem 13 we get that $e_\lambda S$ $(\lambda \in \Lambda)$ are the 0-minimal right ideals of S. So we can write $S = \bigcup_{\lambda \in \Lambda} Se_\lambda = \bigcup_{\lambda \in \Lambda} e_\lambda S$. Hence since $S^2 = S$,

$$S = S^2 = \Big(\bigcup_{\lambda \in \Lambda} e_\lambda S\Big)\Big(\bigcup_{\lambda' \in \Lambda} Se_{\lambda'}\Big) = \bigcup_{\lambda, \lambda' \in \Lambda} e_\lambda S e_{\lambda'}. \tag{10}$$

In view of Lemma 6 $e_\lambda S e_{\lambda'}$ $(\lambda, \lambda' \in \Lambda)$ are quasi-ideals of S satisfying $0 \subseteqq e_\lambda S e_{\lambda'} \subseteqq e_\lambda S \cap Se_{\lambda'}$. This and Theorem 8 imply that $e_\lambda S e_\lambda$ $(\lambda, \lambda' \in \Lambda)$ are either 0 or 0-minimal quasi-ideals of S.

We have to verify only condition 3). Let $e_\lambda S e_{\lambda'} \neq 0$. The product $Se_\lambda S e_{\lambda'}$ is a left ideal $\neq 0$ contained in the 0-minimal left ideal $Se_{\lambda'}$. Hence $Se_\lambda S e_{\lambda'} = Se_{\lambda'}$, whence $e_{\lambda'} S e_\lambda \,.\, e_\lambda S e_{\lambda'} = e_{\lambda'} S e_{\lambda'} \neq 0$.

(D) implies (E). We have only to show the regularity of S. By supposition $S = \bigcup_{\lambda, \lambda' \in \Lambda} \mathfrak{k}_{\lambda\lambda'} = \bigcup_{\lambda, \lambda' \in \Lambda} e_\lambda S e_{\lambda'}$. Let $a = e_\lambda s e_{\lambda'}$ $(\neq 0)$ be an arbitrary element of S.

By 3) the hypothesis $e_\lambda S e_{\lambda'} \neq 0$ implies $e_{\lambda'} S e_\lambda \neq 0$. In view of Lemma 6 the product $e_\lambda s e_{\lambda'} \,.\, e_{\lambda'} S e_\lambda$ is a quasi-ideal of S. The 0-minimality of the quasi-ideal $e_\lambda S e_\lambda$ implies that either $e_\lambda s e_{\lambda'} \,.\, e_{\lambda'} S e_\lambda = 0$ or $e_\lambda s e_{\lambda'} \,.\, e_{\lambda'} S e_\lambda = e_\lambda S e_\lambda$ holds.

The first possibility implies $e_\lambda S e_\lambda s e_{\lambda'} \,.\, e_{\lambda'} S e_\lambda = 0$. Since the quasi-ideal $e_\lambda S e_\lambda s e_{\lambda'}$ $(\neq 0)$ is contained in the 0-minimal quasi-ideal $e_\lambda S e_{\lambda'}$, we get $e_\lambda S e_\lambda s e_{\lambda'} = e_\lambda S e_{\lambda'}$. Thus $e_\lambda S e_{\lambda'} \,.\, e_{\lambda'} S e_\lambda = 0$ holds, in contradiction to condition 3).

Thus we necessarily have $e_\lambda s e_{\lambda'} \,.\, e_{\lambda'} S e_\lambda = e_\lambda S e_\lambda$. This implies the existence of an element $e_{\lambda'} t e_\lambda \in e_{\lambda'} S e_\lambda$ with $e_\lambda s e_{\lambda'} \,.\, e_{\lambda'} t e_\lambda = e_\lambda$. Hence $e_\lambda s e_{\lambda'} \,.\, e_{\lambda'} t e_\lambda \,.\, e_\lambda s e_{\lambda'} = e_\lambda s e_{\lambda'}$, or otherwise $a(e_{\lambda'} t e_\lambda)\, a = a$, which says that a is a regular element of S. This proves our assertion.

(E) implies (A). Corollary 11 and Theorem 14 imply that we can write every 0-minimal quasi-ideal $\mathfrak{k}_\alpha$ $(\alpha \in A)$ of S in the form $\mathfrak{k}_\alpha = \mathfrak{l}_\alpha \cap \mathfrak{r}_\alpha$, where $\mathfrak{l}_\alpha = Se_\alpha$ $(e_\alpha^2 = e_\alpha)$ is a 0-minimal left ideal and $\mathfrak{r}_\alpha = f_\alpha S$ $(f_\alpha^2 = f_\alpha)$ is a 0-minimal right ideal of S. Thus $S = \bigcup_{\alpha \in A} \mathfrak{k}_\alpha = \bigcup_{\alpha \in A} [\mathfrak{l}_\alpha \cap \mathfrak{r}_\alpha] \subseteqq \bigcup_{\alpha \in A} \mathfrak{l}_\alpha \subseteqq S$. Hence $S = \bigcup_{\alpha \in A} \mathfrak{l}_\alpha = \bigcup_{\alpha \in A} Se_\alpha$, q.e.d.

Theorems 4, 5 and 15 imply:

Corollary 16. *The following four conditions on a semigroup S with* 0 *and with more than one element are equivalent*:

(a) *S is an inverse semigroup and the union of its* 0-*minimal left ideals*;

(b) *S is a* 0-*direct union of ideals which are Brandt subsemigroups of S*;

(c) *S is a* 0-*direct union of ideals which are completely* 0-*simple inverse subsemigroups of S*;

(d) *S is an inverse semigroup and the union of its* 0-*minimal quasi-ideals.*

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Резюме

ПОЛУГРУППЫ КОТОРЫЕ ЯВЛЯЮТСЯ ОБЪЕДИНЕНИЕМ ВПОЛНЕ ПРОСТЫХ ПОЛУГРУПП С НУЛЕМ

ОТО ШТЕЙНФЕЛД (Oto Steinfeld), Будапешт

Целью статьи является изучение условий при которых полугруппа с нулем имеет следующее свойство: Она является объединением вполне простых полугрупп, причём пересечение всяких двух отличных компонент — нуль полугруппы.